UDC 624.04:41

Shkurupiy A.A.*, PhD, Professor*
ORCID 0000-0003-1487-1037 Shkurupiy.AA@gmail.com
Mytrofanov P.B.*, PhD, Associate Professor*
ORCID 0000-0003-4274-1336 Mytrofanov.P@gmail.com
Poltava National Technical Yuri Kondratyuk University

INFLUENCE OF LONGITUDINAL FORCES FOR DETERMININGTHE FREQUENCY OF FREE OSCILLATIONS DISCRETE DYNAMIC SYSTEMS

In the research flat and spatial rod system as a part of many engineering and technical structures were considered. A new construction with a simultaneous increase quality of their design requires finding new ways to improve dynamic analysis and calculation of complex rod systems. It was analyzed, that longitudinal forces consideration when determining discrete dynamical systems free oscillations frequency together with use of modern computer hardware and software systems allows to reduce time for such calculations, and use the innovative schemes and methods that were previously inaccessible through large amounts of computing. The problem of road systems free oscillations frequencies with finite number of freedom degrees in bending with considering of longitudinal forces was considered. The examples of quantitative assessment influence of longitudinal forces on the frequencies of free oscillations were shown.

Keywords: *free oscillations, longitudinal force, frequency, dynamic system, load.*

Шкурупій О.А.*, к.т.н., професор*
Митрофанов П.Б.*, к.т.н., доцент*
Полтавський національний технічний університет імені Юрія Кондратюка

УРАХУВАННЯ ПОЗДОВЖНІХ СИЛ ПРИ ВИЗНАЧЕННІ ЧАСТОТ ВІЛЬНИХ КОЛИВАНЬ ДИСКРЕТНИХ ДИНАМІЧНИХ СИСТЕМ

Розглянуто плоскі й просторові стержневі системи, що входять до складу багатьох інженерних і технічних споруд. Скорочення строків розробки нових конструкцій з одночасним підвищенням якості їх проектування потребує знаходження нових шляхів для удосконалення динамічного аналізу й розрахунку складних стержневих систем. Проаналізовано, що урахування поздовжніх сил при визначенні частот вільних коливань дискретних динамічних систем сумісно з використанням сучасної комп'ютерної техніки і програмних комплексів дозволяє зменшити час для проведення таких розрахунків, а також використовувати принципово нові схеми та методи, які були недоступні раніше через великі обсяги обчислень. Розглянуто задачу обчислення спектра частот вільних коливань стержневих систем із кінцевим числом ступенів вільності при згинанні з урахуванням поздовжніх сил. Наведено приклади кількісної оцінки впливу поздовжніх сил на частоти вільних коливань.

Ключові слова: *вільні коливання, поздовжня сила, частота, динамічна система, навантаження.*

Introduction. The article contains results of theoretical research of longitudinal forces quantitative assessment influence in the spectrum of dynamic rod systems free oscillations frequencies with a finite number of freedom degrees in bending. The results of quantitative assessment impact forces in the longitudinal frequency spectrum free oscillation in bending shown as graphs depending on the free oscillations frequency of the longitudinal forces values.

Analysis of quantitative assessment influence of longitudinal forces in the spectrum of free oscillations frequencies in bending shows that longitudinal forces significantly alter the frequency of free oscillations (compressive – reduce the frequency and stretching – increase them). Longitudinal forces also took position on the frequency spectrum.

It is proved that the calculation of the free oscillations frequency spectrum must consider the influence of longitudinal forces.

Review of recent sources of research and publications. It is known that the physical properties of real structures exactly display discrete-continuous models, for which there is no comprehensive approach that allows with desirable accuracy and minimal expenses perform their full dynamic calculation [1, 2]. Additional difficulties arise when calculating spatial branching structures, they are connected with the necessity of modeling compatible oscillations, considering the density of the frequency spectrum and the influence of different factors: the transverse and longitudinal loads, type of concentrated impurities, shear median surface, deplanation and turn sections, damping, elastic links type and other boundary conditions. Even with modern computing tools and technologies many dynamic problems remain inaccessible for direct solution [3 – 5].

Parts of the common problems unsolved earlier. It is known that when calculating linear deformed rod systems with a finite number of freedom degrees on small free oscillations for calculating frequencies spectrum, frequency equation is solved [2 – 7]. The coefficients of this equation are expressed through relevant elements of matrix stiffness or pliability system. Usually elements of these matrices are determined by undeformed system scheme and depend on its mechanical properties. When calculating system by the deformed scheme manifested mutual influence of certain types deformations; for example, when the rod bent transverse load longitudinal forces causing an additional bending. This leads to the fact that the stiffness will depend on the longitudinal forces, and accordingly, free oscillations frequencies of such a dynamic system also depend on the longitudinal load.

Purpose of the work. To quantify the influence of longitudinal forces in the spectrum of dynamic rod systems free oscillations frequencies with finite number of bending freedom degrees.

Main material and results. Consider free oscillations of the cantilevered beam with a point mass m at its end (Figure 1, a). (EI = const; EA = const).

If the longitudinal force is zero, then the circular free oscillations frequency without considering the resistance forces ω was calculated by the formula

$$\omega = \frac{1}{\sqrt{m\delta_{11}}}, \tag{1}$$

where $\delta_{11} = l^3/3EI$ (Figure 1, b).

In the case where $N \neq 0$, calculate displacement δ_{11}^{c} from $F = 1$ (when $N<0$ – compression, Figure 1, c) and displacement δ_{11}^{p} (when $N > 0$ – stretching).

The corresponding differential equations of curved beam axis shall be:

$$-\mathit{for}\ N<0, \qquad EIy'' = -Ny + \delta_{11}^{c} N + (\ell - x); \tag{2}$$

$$-\mathit{for}\ N>0, \qquad EIy'' = Ny - \delta_{11}^{p} N + (\ell - x). \tag{3}$$

Denote $k^2 = N/EI$ and write the general solution for (2 and 3) in the form:

$$-for\ N<0, \qquad y=-\left(\frac{\ell+N\delta_{11}^{c}}{N}\right)\cos kx+\frac{1}{kN}\sin kx+\frac{(\ell-x)}{N}+\delta_{11}^{c}; \qquad (4)$$

$$-for\ N>0, \qquad y=\left(\frac{\ell+N\delta_{11}^{p}}{N}\right)chkx-\frac{1}{kN}shkx-\frac{(\ell-x)}{N}+\delta_{11}^{p}. \qquad (5)$$

For determination of the necessary displacements δ_{11}^{c} i δ_{11}^{p} it used use conditions: when $x=\ell$ and $N<0$ $y=\delta_{11}^{c}$; when $x=\ell$ and $N>0$ $y=\delta_{11}^{p}$.

Then from the equations (4) and (5) it is obtained:

– when $N<0$ $$\delta_{11}^{c}=\frac{\ell^{3}}{3EI}\frac{3(tgk\ell-k\ell)}{(k\ell)^{3}}; \qquad (6)$$

– when $N>0$ $$\delta_{11}^{p}=\frac{\ell^{3}}{3EI}\frac{3(k\ell-thk\ell)}{(k\ell)^{3}}. \qquad (7)$$

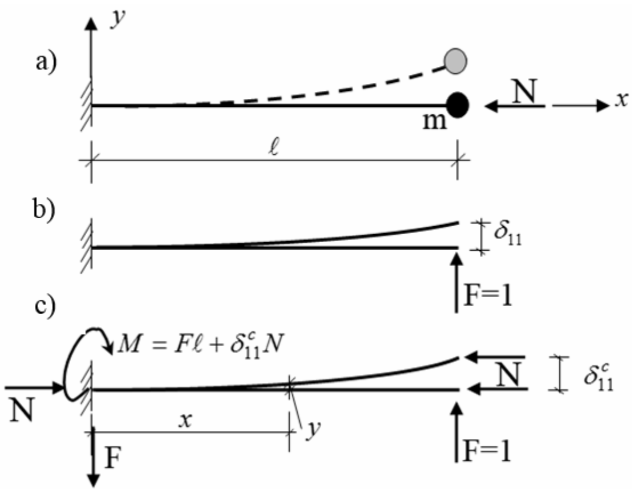

Figure 1 – Design scheme of the beam

Thus, when calculating free oscillations frequency considering longitudinal forces in the formula (1) instead δ_{11} it needs to substitute values calculated by formulas (6) and (7).

The results of displacements calculation δ_{11}^{c}, δ_{11}^{p} and also the frequencies ratio ω^{c}/ω, ω^{p}/ω are shown in figures 2 and 3.

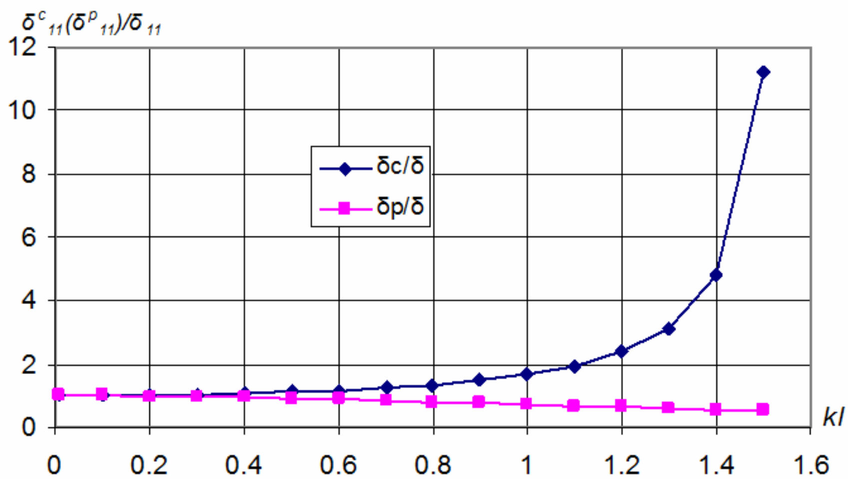

Figure 2 – Displacements δ_{11}^{c} and δ_{11}^{p}, calculated by the formula (6, 7) when $l^{3}/(3EI)=1$

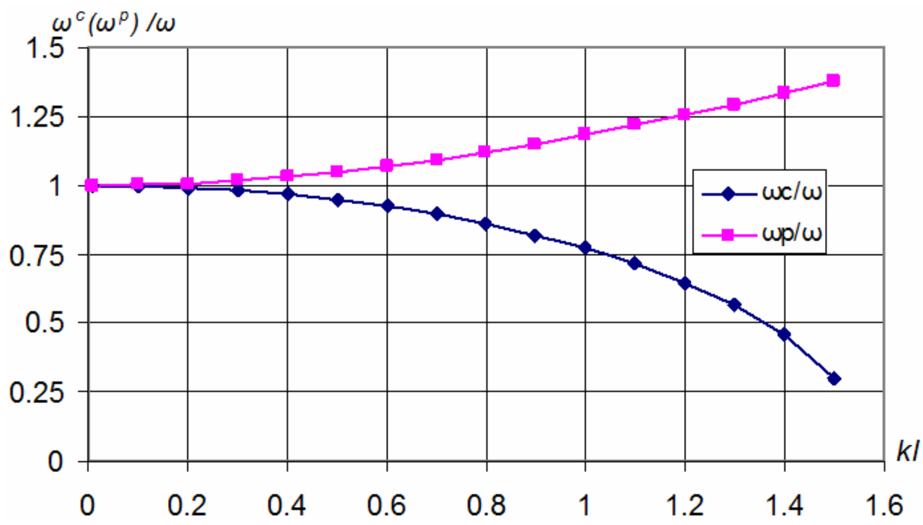

Figure 3 – The dependence of frequencies ratios ω^c/ω and ω^p/ω from *kl*

The dependence of frequency ω^c/ω on the ratio N/N_{cr} isillustrated in figure 4.

As can be seen from the graphics the influence of longitudinal compressing forces on the frequency of free oscillations is less than 5% to the ratio N/N_{cr} = 0,1. With increasing longitudinal forces, frequency reduced significantly.

For example, when N/N_{cr} = 0,58, ω^c/ω=0,65.

Consider the dynamic rod system with two degrees of freedom (figure 5).

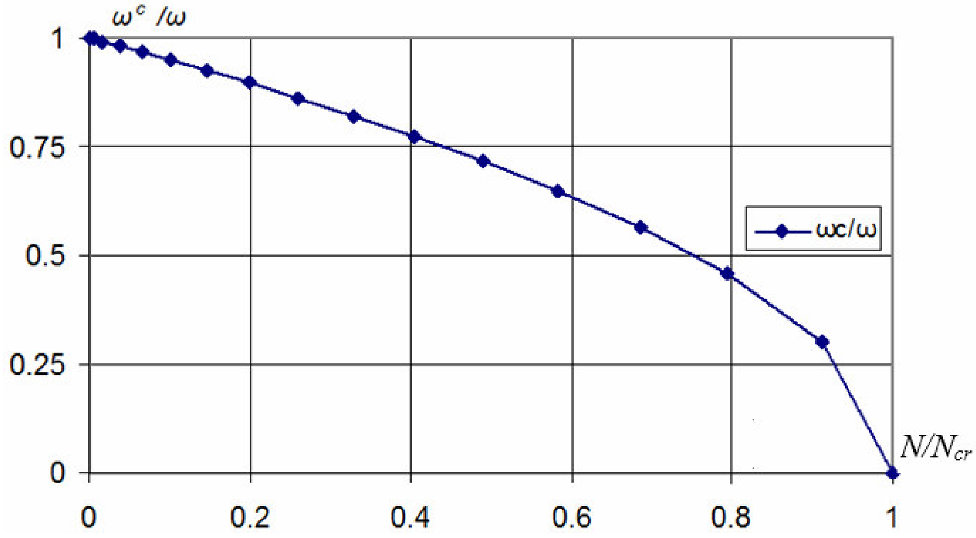

Figure 4 – The dependence of frequencies ratios ω^c / ω on the ratio N/N_{cr}

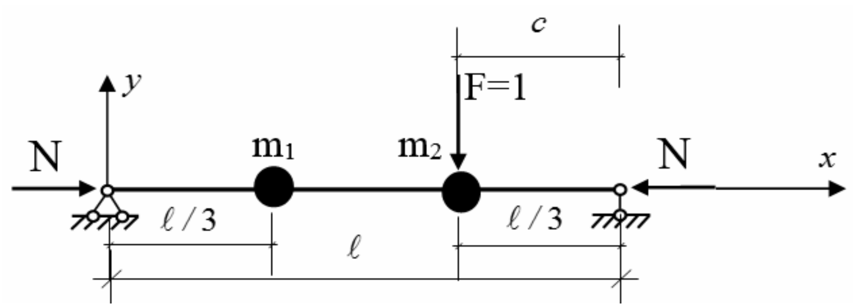

Figure 5 – Design scheme of the dynamic system ($m_1 = m_2 = m$)

Differential equations of curved beam axis (figure 5) have the form:

– for the area to the left of the force F when N<0 ($0 \le x \le \ell - c$)

$$EIy'' = -Ny - Fcx / \ell \; ; \tag{8}$$

– for the area to the right of the force F when N<0 ($\ell - c \le x \le \ell$)

$$EIy'' = -Ny - F(\ell - c)(\ell - x) / \ell \; ; \tag{9}$$

After solving the system of equations (8) and (9) it is obtained the equation of the bent axis which to the left of forces *F* (figure 5) has form [3]

$$y = \frac{F \cdot \sin(kc)}{Nk \cdot \sin(k\ell)} \sin(kx) - \frac{F \cdot c}{N \cdot \ell} x \; , \tag{10}$$

on condition that $k^2 = N / EI$.

Equation (10) allows to calculate displacement in the points of the dynamic system mass location from the *F=1*.

Considering that the dynamic system is symmetric, it is got:

when $x = 2l/3$, $c = l/3$, $\delta_{22} = \delta_{11}$,

when $x = l/3$, $c = l/3$, $\delta_{12} = \delta_{21}$.

Thus, the expressions for these movements will be:

$$\delta_{11} = \delta_{22} = \frac{F \cdot sin(k\ell/3)}{Nk \cdot sin(k\ell)} sin\left(\frac{2k\ell}{3}\right) - \frac{2F\ell}{9N} = \frac{F \cdot \ell}{N}\left(\frac{9 \cdot sin(k\ell/3) \cdot sin(2k\ell/3) - 2k\ell \cdot sin(k\ell)}{9k\ell \cdot sin(k\ell)}\right), \tag{11}$$

$$\delta_{12} = \delta_{21} = \frac{F \cdot sin(k\ell/3)}{Nk \cdot sin(k\ell)} sin(k\ell/3) - \frac{F \cdot \ell}{9N} = \frac{F \cdot \ell}{N}\left(\frac{9 \cdot sin(k\ell/3) \cdot sin(k\ell/3) - k\ell \cdot sin(k\ell)}{9k\ell \cdot sin(k\ell)}\right) . \tag{12}$$

$$\frac{F\ell}{N} = \frac{F\ell}{k^2 EI} = \frac{1 \cdot \ell^3}{(k\ell)^2 EI} \; .$$

When $k\ell = 0$, $\delta_{11} = \delta_{22} = \frac{8}{486} \cdot \frac{\ell^3}{EI}$, and $\delta_{12} = \delta_{21} = \frac{7}{486} \cdot \frac{\ell^3}{EI}$.

The results of calculating frequency spectrum ω_1 i ω_2 depending on $k\ell$ and their ratio ω_1 / ω_2 are shown on figures 6, 7 and 8 respectively.

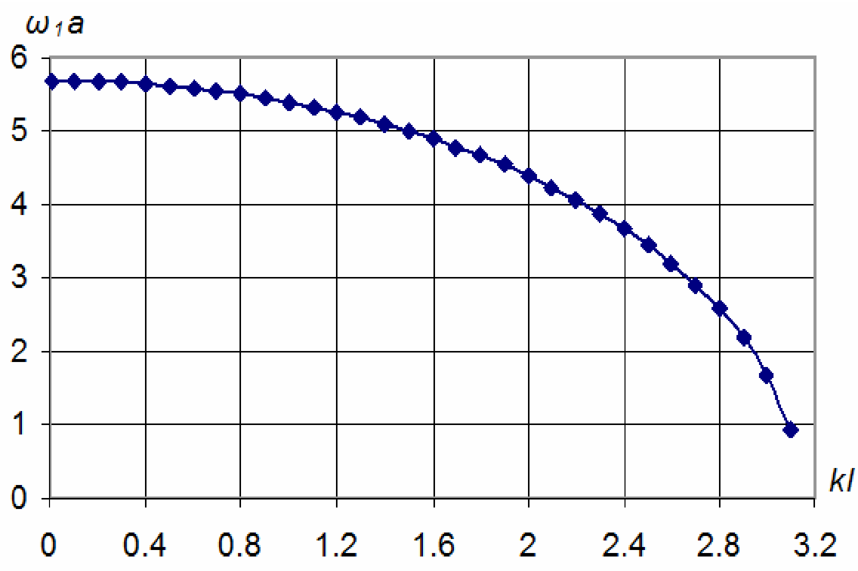

Figure 6 – The dependence of frequency ω_1 from $k\ell$ when $a = \sqrt{m\ell^3 /(EI)}$

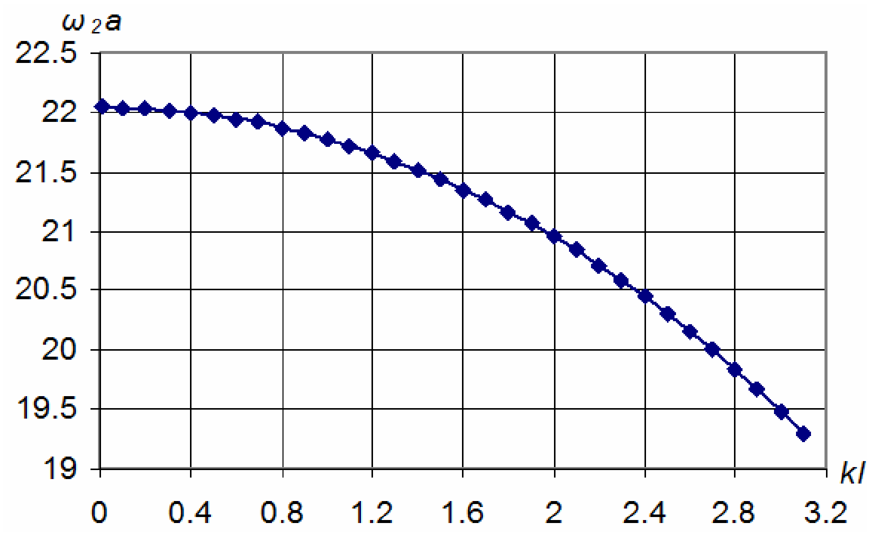

Figure 7 – The dependence of frequency ω_2 from $k\ell$ when $a = \sqrt{m\ell^3 /(EI)}$

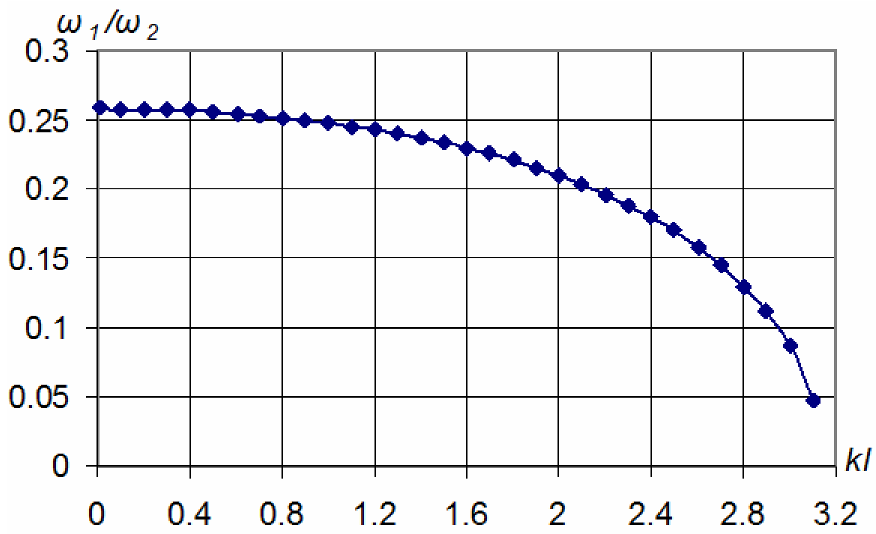

Figure 8 – The dependence of frequency ratio ω_1 / ω_2 from $k\ell$

The dependence of frequency ω_1 on the ratio N/N_{cr} is illustrated in figure 9.

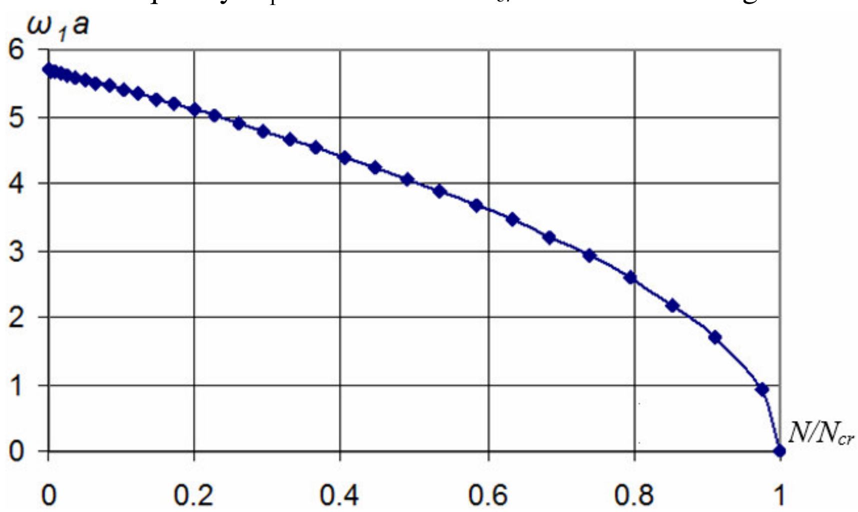

Figure 9 - The dependence of frequency $\omega_1 a$ on the ratio N/N_{cr} when $a = \sqrt{m\ell^3 /(EI)}$

As can be seen from the graphics, the influence of longitudinal compressing forces on fundamental tone frequency is less than 5% to the ratio N/N_{cr}=0.1. With increasing longitudinal forces, frequency significantly reduced.

For example, when $N/N_{cr}=1\cdot10^{-5}$ frequency $\omega_1 = 5{,}692/a$, and when N/N_{cr}=0,58 frequency decreased to $\omega_1 = 3{,}675/a$, which is 35,4%.

Conclusions. Analysis of longitudinal forces quantitative estimation influence on frequency spectrum of dynamic rod systems free oscillations with finite number of freedom degrees at bending are showed. Longitudinal forces considerably change the frequency of free oscillations (see figures 3, 4, 6, 7 and 9). Compressive forces reduce the frequency of free oscillations but stretching forces increase them. Longitudinal forces also are changing frequencies position on the frequency spectrum of free oscillations (see figure 8).

References

1. *Тимошенко С. П. Курс теории упругости / С. П. Тимошенко. – К. : Наукова думка, 1972. – 501 с.*
   Timoshenko S. P. Kurs teorii uprugosti / S. P. Timoshenko. – K. : Naukova dumka, 1972. – 501 s.
2. *Structural Mechanics / H. Al. Nageim, F. Durka, W. Morgan, D. Williams. – Trans-Atlantic Publications, 2010. – 456 p.*
3. *Olsson K.-G. Structural Mechanics: Modelling and Analysis of Frames and Trusses / Karl-Gunnar Olsson, Ola Dahlblom. – Wiley, 2016. – 344 p.*
4. *Smith P. An Introduction to Structural Mechanics / P. Smith. – Palgrave Macmillan, 2001. – 368 p.*
5. *Hulse R. Structural Mechanics / Ray Hulse, Jack Cain. – Palgrave Macmillan. 2000. – 592 p.*
6. *Бабаков Н. М. Теория колебаний / Н. М. Бабаков. – М. : Гос. изд-во технико-теоретич. лит., 1958. – 628 с.*
   Babakov N. M. Teorija kolebanij / N. M. Babakov. – M. : Gosud. izd-vo tehnikoteoretich. lit., 1958. – 628 s.
7. *Строительная механика. Динамика и устойчивость сооружений / А. Ф. Смирнов, А. В. Александров, Б. Я. Лащенников, Н. Н. Шапошников; под ред. А. Ф. Смирнова. – М. : Стройиздат, 1984. – 416 с.*
   Stroitel'naja mehanika. Dinamika i ustojchivost' sooruzhenij / A. F. Smirnov, A. V. Aleksandrov, B. Ja. Lashhennikov, N. N. Shaposhnikov; pod red. A. F. Smirnova. – M. : Strojizdat, 1984. – 416 s.

Received 22.02.2017