https://doi.org/10.26906/znp.2018.50.1061

UDC 624.074.433

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THIN-WALLED BARRELL SHELL DEFLECTIVE MODE ANALYSIS

The paper deals with the deflective mode of steel rotary shell with different form of outer surface that are loaded with axially symmetric load. The results show solution of shell voltage and strain equation under the load that is described by exponential law and based on efforts from temperature differentials. Besides, the paper represents design formulas for deflection analysis, running bending moments and running transverse forces in shells with different abutment to the basis. It was shown the basic function of deflection and maximum value of relevant parameters of the reaction. The analysis of aspects about efficiency of using corrugated wall for steel barrel shell is done. The results in analytical and graphical form are shown. According to resulting formulas, it was made comparative calculations of shells with constant and variable wall thickness.

***Keywords:** barrel shell, exponential law, deflection functions, state of stress, zone of end effect.*

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АНАЛІЗ НАПРУЖЕНО-ДЕФОРМОВАНОГО СТАНУ ТОНКОСТІННИХ ЦИЛІНДРИЧНИХ ОБОЛОНОК

Стаття присвячена дослідженню напружено-деформованого стану сталевих оболонок обертання, з різною формою зовнішньої поверхні, звантажених осесиметричним навантаженням. Отримане рішення рівняння напружень і деформацій оболонки при навантаженні, що описується експоненціальним законом та з урахуванням зусиль від перепаду температури. Зокрема, представлені розрахункові формули для визначення прогинів, погонних згинальних моментів та погонних поперечних сил в оболонках з різним примиканням до основи. Приведені основні функції прогинів, а також максимальні значення відповідних параметрів реакції. Виконаний аналіз аспектів, щодо ефективності застосування гофрованої стінки для сталевих циліндричних оболонок. Результати представлені в аналітичному та графічному вигляді. Відповідно до отриманих формул, виконані порівняльні розрахунки оболонок з постійною та змінною товщиною стінки.

***Ключові слова:** циліндрична оболонка, експоненціальний закон, функції прогинів, напружений стан, зона крайового ефекту.*

Introduction. Steel rotary shell is a prototype of many real constructions. In particular it is the prototype of cylindrical capacities for keeping bulk material which exploration was in the last authors' works. In this paper barrel shells are examined from this point, and it is the vector to be researched. But the resulting aspects can be used in other branches of material durability and construction elements.

The analysis of the last researches and publications. A lot of national and foreign scientists explore the deflective mode of barrel shells [1 – 10]. Though the named sources are purely theoretical and the given formulas are not appropriate enough for practical solving of calculation and designing capacities for keeping bulk materials.

Parts of the general problem that were not researched earlier. The determination of the internal efforts and displacement of the thin-walled barrel shell on different boundary conditions is shown in many scientific and reference sources. However, there must be pointed out particular concepts that were not examined yet. First of all, it is to find a solution under the load. It is described by the exponential law that is typical for bulk material pressure. Secondly, there is no analysis of aspects about efficiency of using corrugated plate for the shell surface. The achieved results are useful for further examination of shells that are supported by upright stiffening rib.

Problem statement. The general aim of research is the solution of shell voltage and strain equation under the load that is described by exponential law, and derivation of practical formulas for deflection analysis, running bending moments and running transverse forces in shells with different abutment to the basis.

Main materials and results. The shell with the diameter D_w and the height H_w is loaded with axially symmetric load *p(x)*. Independent variable x indicates that radial load *p(x)* in general case can be changeable in length or function on the bounded length of shell. To determine voltage and strain of such shell, there should be solved the differential equation [1, 2, 9, 10]

$$\frac{d^4w(x)}{dx^4} + 4k_w^4(x) = \frac{p(x)}{D_r} , \qquad (1)$$

where D_r is the cylindrical stiffness of the shell on flexion in circular direction; $w(x)$ is the function of the shell body displacement.

Coefficient k_w can be defined using the equation

$$k_w^4 = Et_{ef,r} \Big/ \left(D_w^2 D_r\right), \qquad k_w^4 = Et_w \Big/ \left(D_w^2 D_r\right), \qquad (2)$$

where $t_{ef,r} = t_w \ell_{w,1} / \ell_w$ is the thickness of plates (stiffness of its equivalent to the stiffness of the corrugated profile with the thickness t_{ef}) that receive circular efforts; $l_{w,1}$ is the length of corrugation sweep on the corrugation plate of shell l_w; E is the elastic modulus of the material.

Equation (1) is used for the shell with the constant thickness t_w regardless of the surface form. It can be both smooth and corrugate. The used simplification about constancy of thickness substantiates that numerical results solving of the differentiate equation (1) with the changeable thickness nearly do not differ from solving when t_w = const if to assume the thickness of wall as the end of the shell.

By using trigonometric function, the solution for the differentiate equation (1) is the next

$$w(x) = e^{-k_w x}\left[C_1 \sin(k_w x) + C_2 \cos(k_w x)\right] + e^{k_w x}\left[C_3 \sin(k_w x) + C_4 \cos(k_w x)\right] + w_* , \qquad (5)$$

where C_1, C_2, C_3 and C_4 are constant integrations, that are defined by boundary conditions, and $w_*(x)$ is the partial solution of the differentiate equation, that is defined by analytical form of putting down the function *p(x)*.

As it is know, deflection of shell that is defined by equation (5) is the two pairs of rapidly decayed periodical functions. Each of them decays depending on the distance from the top or lower edge. For the shells long enough that are met the demands of membrane theory of shells, both parts of equation for deflection have an independent meaning. The first part describes image of strained condition near the shell surface, and the second – on the top edge. Considering this feature and considering further shells pinched near the surface, it can be considered that C_3 and C_4 are equal to zero.

Considering two variants of solution, when $p(x) = p_0$ and $p(x) = p_0(1-\alpha e^{\beta x})$, it can be conformed the famous Yansen-Kenen's formula. In the first case partial solution $w_d(x)$ has an easy solve such as

$$w_* = w_0 = p_0 / (4k_w^4 D_r), \tag{6}$$

where w_0 is an introduced designation for this equation.

Constant integrations on condition are defined, when $x = 0$ the deflection and rotation angle are equal to zero. After a number of calculations there is

$$w(x) = w_0 \left\{1 - e^{-k_w x}\left[\sin(k_w x) + \cos(k_w x)\right]\right\}, \tag{7}$$

It must be pointed out that deflection of shells from the shaped sheet will be less only on ΔA_w quantity than the shells from the flat sheet (area ratio of cross-sections of shaped and flat sheets). Since the given quantity is not very different from the unit, shaping of sheets during the axially symmetric load has a quite indirect value. During the load $p(x) = p_0(1-\alpha e^{\beta x}) = p_0 K_p(x)$ partial solution of differential equation (1) appropriate to solve in the form of

$$w_*(x) = w_0\left(1 - \frac{\alpha\, e^{\beta x}}{1 + 0.25\zeta^4}\right), \tag{8}$$

where $\zeta=\beta/k_w$ is the ratio of the congruent coefficient.

General solution of the differentiate equation is quite cumbersome

$$\begin{gathered} w(x) = w_0 \left\{1 - e^{-k_w x}\left[\sin(k_w x) + \cos(k_w x)\right]\right\} - \\ -w_0 \frac{\alpha e^{\beta x}}{(1 + 0.25\zeta^4)}\left\{1 - e^{-k_w x(1+\zeta)}\left[(1+\zeta)\sin(k_w x) + \cos(k_w x)\right]\right\}. \end{gathered} \tag{9}$$

Since for every sort of agricultural production the coefficient ζ is quite little, therefore ζ^4 acquire lesser value. It allows to equal this coefficient to zero and to put down the formula (9) in easier form

$$w(x) = w_0 K_p(x)\left\{1 - e^{-k_w x}\left[\sin(k_w x) + \cos(k_w x)\right]\right\}, \tag{10}$$

For levels x that are detached from the shell surface, the quantity indicated in braces can be neglected and rate deflections using the equation

$$w(x) = w_0 K_p(x). \tag{11}$$

Parameters p_0, α and β have the next value in problems of barrel shells calculation

$$p_0 = \gamma_g D_w / 4 f_g, \qquad \beta = 4\lambda_0 f_g / D_w, \qquad \alpha = e^{-\beta H_w}, \tag{12}$$

where γ_g, f_g and λ_0 are the calculated value of specific gravity, frictional coefficient and coefficient of lateral pressure for food storage.

It should be defined how far the levels x have to be from the shell surface and to be analyzed the effect from the shaping sheets on the given quantity. There should be performed the rate of decay speed of functions (7) and (9) with the help of crest value ratio of deflections in the range of two adjacent half-waves. Let the length of half-wave be line $\lambda = \pi / k_w$. For the cases of constant and exponent load the decay speed is characterized by the quality $e^{\lambda k_w}$

(for exponential load $e^{\lambda k_w(1-\zeta)} \approx e^{\lambda k_w}$), i. e. top levels for which $x > \pi / k_w$ can be considered as distant enough from the surface. Considering that shells, consisting of shaped sheets, have the coefficient value k_w it is lesser than the shells with flat wall. That is why it has longer zone of end effect in k_λ times

$$k_\lambda = \sqrt[4]{\frac{D_r}{D_0}\frac{t_w}{t_{ef}}} \ , \tag{13}$$

where value D_0 is defined as $D_0 = Et_w^3 \Big/ \left[12(1-\mu^2)\right]$.

Perform the rate of internal efforts using the classic ratios of materials strength:

• circular normal strain

$$\sigma_h(x) = 2w(x)\frac{E}{D_w}; \tag{14}$$

• running bending moments

$$M_x(x) = -D_r\frac{d^2w(x)}{dx^2}; \tag{15}$$

• running transverse forces

$$Q_x(x) = \frac{dM_x(x)}{dx} = -D_r\frac{d^3w(x)}{dx^3}. \tag{16}$$

The received analytic dependences have been summarized to the Table 1, where apart from the equations for bending moments and transverse forces, the main functions of deflection and maximum value of the corresponding parameters of reaction are given.

The procedure of internal force factor receiving has to be studied additionally when the shell has a swing joint to the surface. Results in accurate mathematical statements for $M_x(x)$ and $Q_x(x)$, when the exponent load operates, are quite cumbersome. But the used simplification about lesser coefficient ζ allows significantly simplify final equations. Attention to important feature of received dependences should be paid to. If the quantity of shell deflections from flat and shaped sheets is practically not different, this assertion is not true for value of bending moments and transverse forces.

For shells performed from shaped sheets, bending moments will be bigger in k_λ^2 times, and transverse forces – in k_λ^4 times. It is explicated by enlarged length of end effect zone that in k_λ times lesser in shells with the flat wall than with the shaped. More clearly the given feature is illustrated on the Drawing 1.

I should be concentrated on one more aspect of using the differentiate equation (1). Respectively to norms of design, such as NBS (National Building Standards) [12] and Eurocode [13], spectrum of efforts from axially symmetric pressure of bulk material is always added by efforts from temperature differential Δt. Considering that this differential does not depend on height x, the additional deflections are equal to $w_t = 0.5\alpha_t D_w \Delta t$, where α_t is the coefficient of temperature expansion.

But the given deflection can be induced by some uniformly distributed load p_t over girth and height of the shell that is connected with w_t ratio

$$w_t = p_t D_w^2 / (4Et_w) . \tag{17}$$

Probably the density of load

$$p_t = 2\alpha_t \Delta t E \frac{t_w}{D_w}. \tag{18}$$

Table 1 – Design equations for determination of deflections and internal efforts in shells with different abutment to the basis

№	Equation *R(x)*	x_{max}	R_{max}
The shell stiffened near the surface (uniform load)			
1	$w(x)=w_0\left\{1-e^{-k_w x}\left[\sin(k_w x)+\cos(k_w x)\right]\right\}$	λ	$w_{\max}=1.043w_0$
2	$M_x(x)=\dfrac{p_0}{2k_w^2}e^{-k_w x}\left[\cos(k_w x)-\sin(k_w x)\right]$	0	$M_{x,\max}=0.5\dfrac{p_0}{k_w^2}$
3	$Q_x(x)=\dfrac{p_0}{k_w}e^{-k_w x}\left[1-2\sin^2\left(\dfrac{k_w x}{2}\right)\right]$	0	$Q_{x,\max}=\dfrac{p_0}{k_w}$
The shell stiffened near the surface (exponential load)			
4	$w(x)=w_0K_p(x)\left\{1-e^{-k_w x}\left[\sin(k_w x)+\cos(k_w x)\right]\right\}$	λ	$w_{\max}=1.043w_0K_p(\lambda)$
5	$M_x(x)=\dfrac{p_0}{2k_w^2}K_p(x)e^{-k_w x}\left[\cos(k_w x)-\sin(k_w x)\right]$	0	$M_{x,\max}=0.5\dfrac{p_0}{k_w^2}(1-\alpha)$
6	$Q_x(x)=\dfrac{p_0}{k_w}K_p(x)e^{-k_w x}\left[1-2\sin^2\left(\dfrac{k_w x}{2}\right)\right]$	0	$Q_{x,\max}=\dfrac{p_0}{k_w}(1-\alpha)$
The shell with a swing joint to the surface (uniform load)			
7	$w(x)=w_0\left[1-e^{-k_w x}\cos(k_w x)\right]$	$\dfrac{3}{4}\lambda$	$w_{\max}=1.067w_0$
8	$M_x(x)=\dfrac{p_0}{2k_w^2}e^{-k_w x}\sin(k_w x)$	$\dfrac{1}{4}\lambda$	$M_{x,\max}\approx 0.161\dfrac{p_0}{k_w^2}$
9	$Q_x(x)=\dfrac{p_0}{2k_w}e^{-k_w x}\left[\sin(k_w x)-\cos(k_w x)\right]$	0	$Q_{x,\max}=\dfrac{p_0}{2k_w}$
The shell with a swing joint to the surface (exponential load)			
10	$w(x)=w_0K_p(x)\left[1-e^{-k_w x}\cos(k_w x)\right]$	$\dfrac{3}{4}\lambda$	$w_{\max}=1.067w_0K_p\left(\dfrac{3}{4}\lambda\right)$
11	$M_x(x)=\dfrac{p_0}{2k_w^2}K_p(x)e^{-k_w x}\sin(k_w x)$	$\dfrac{1}{4}\lambda$	$M_{x,\max}\approx 0.161\dfrac{p_0}{k_w^2}(1-\alpha)$
12	$Q_x(x)=\dfrac{p_0}{2k_w}K_p(x)e^{-k_w x}\left[\sin(k_w x)-\cos(k_w x)\right]$	0	$Q_{x,\max}=\dfrac{p_0}{2k_w}(1-\alpha)$

Thus, if to consider that $p_0=p_t$, then we can use all the results that were obtained earlier for the load $p(x)=p_0$. Using reference data of NBS (National Building Standards) [14] we will obtain quantitative assessment of the quantity p_t for steel shells. Before this, we have to put down equation (18) in easy and convenient form

$$p_t\approx 5t_w\frac{\Delta t}{D_w}. \tag{19}$$

There should be considered the variant of solving differential equation (1) in the case when the thickness of the shell $t_{ef,r}$ is changing over the height x and find out how functional dependence $t_{ef}(x)$ influence the deflective mode of the shell in comparison with the case when $t_{ef,r}=\text{const}$.

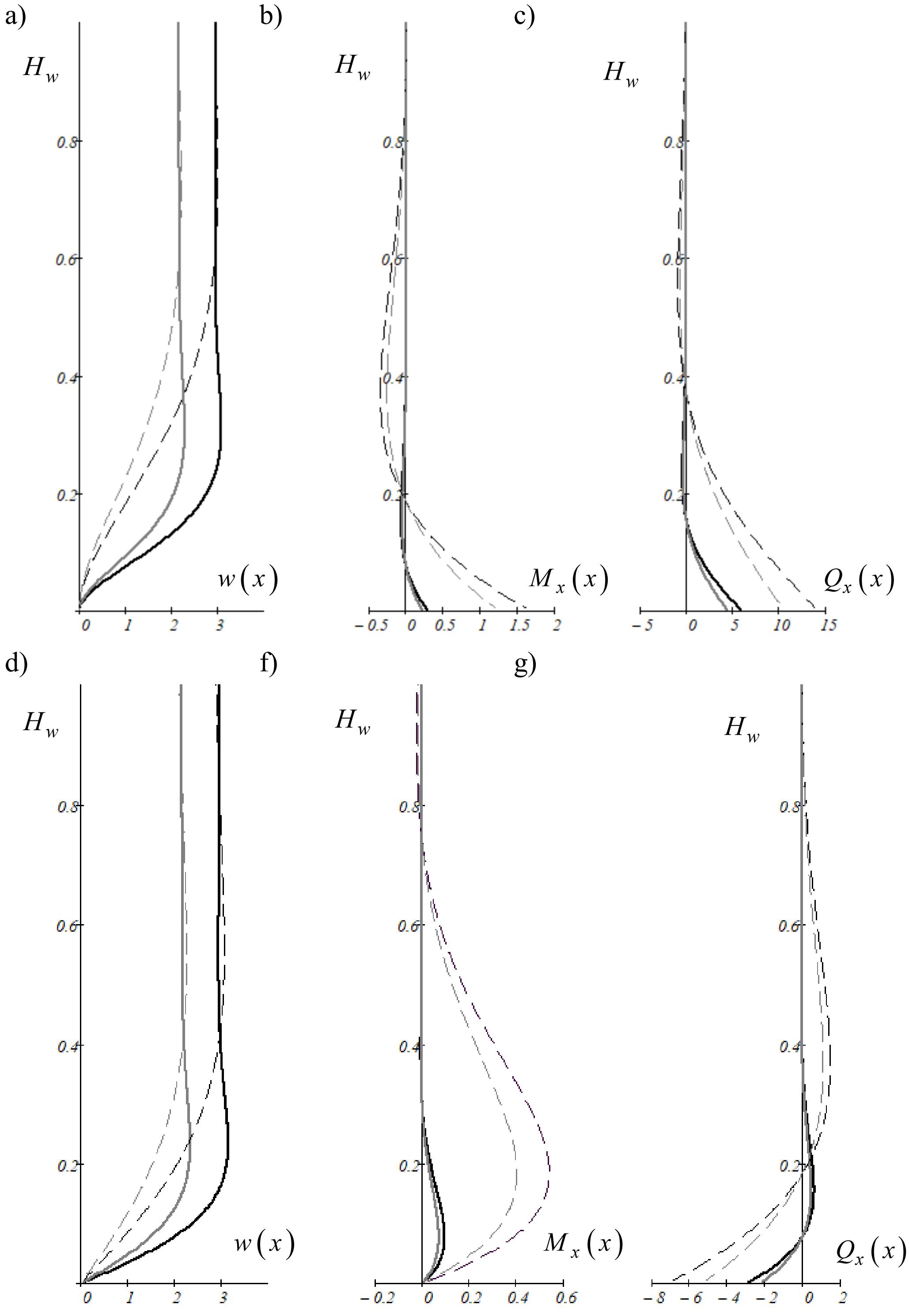

Figure 1 – Charts of deflections (a), running bending moments (b) and running transverse forces (c) for shell stiffened near the surface and with a swing joint to the surface (respectively d, f, g): solid line – flat wall; dotted line – corrugated wall; black line – uniform load; gray line - exponential load

The law of the thickness change over the height is considered in the form of power dependence

$$t_{ef}(x) = t_{ef,r}\, g_w(x) = t_{ef,r}\exp\left[-\varepsilon_w\left(\frac{x}{H_w}\right)\right], \tag{20}$$

where $t_{ef,r}$ is the thickness of shells near the surface; ε_w is non-dimensional parameter that is responsible for the form of curve $g_w(x)$.

Since, the cylindrical stiffness D_r of the shell is the function of height x, the equation (1) must be put down in more general form [11]

$$\frac{d^2}{dx^2}\left[D_r(x)\frac{d^2w(x)}{dx^2}\right] + \frac{4Et_{ef}(x)}{D_w^2}w(x) = p(x), \tag{21}$$

where $D_r(x)$ is the function of cylindrical stiffness of shell that accordingly to equation (20) can be represented in form of the product $D_r(x) = D_r g_w^k(x)$; for shells with flat wall $k = 3$, and with corrugated wall $k = 1$.

Considering the equation for $D_r(x)$, the differentials of equation are revealed (21)

$$\left[g_w^k(x)\frac{d^4w(x)}{dx^4} + g_w(x)\right] + 4k_w^4 g_w(x)w(x) = p(x)/D_r \tag{22}$$

where $f_w(x)$ is additional

$$f_w(x) = 2\frac{dg_w^k(x)}{dx}\frac{d^3w(x)}{dx^3} + \frac{d^2g_w^k(x)}{dx^2}\frac{d^2w(x)}{dx^2}. \tag{23}$$

Using equation (20), there is set the equation (22) more specific form. Respectively for values $k = 1$ and $k = 3$ it is

$$\frac{d^4w(x)}{dx^4} - 2\frac{\varepsilon_w}{H_w}\frac{d^3w(x)}{dx^3} + \frac{\varepsilon_w^2}{H_w^2}\frac{d^2w(x)}{dx^2} + 4k_w^4 w(x) = \frac{1}{g_w(x)}\frac{p(x)}{D_r}, \tag{24}$$

$$g_w^2(x)\left[\frac{d^4w(x)}{dx^4} - 6\frac{\varepsilon_w}{H_w}\frac{d^3w(x)}{dx^3} + 9\frac{\varepsilon_w^2}{H_w^2}\frac{d^2w(x)}{dx^2}\right] + 4k_w^4 w(x) = \frac{1}{g_w(x)}\frac{p(x)}{D_r}. \tag{25}$$

Differential equation (24) is simpler and its solution can be represented in the form that is similar to equation (5). Before making a general solution, take into consideration that the root of characteristic equation (24) is equal

$$\frac{\varepsilon_w}{2H_w}\left[1 \pm \sqrt{1 \pm 8i\left(\frac{k_wH_w}{\varepsilon_w}\right)^2}\right]. \tag{26}$$

Since for the barrel shells $k_wH_w/\varepsilon_w >> 1$

$$k_w\left(1 + \frac{\varepsilon_w}{2H_wk_w}\right) \pm ik_w \approx k_w \pm ik_w, \qquad -k_w\left(1 - \frac{\varepsilon_w}{2H_wk_w}\right) \pm ik_w \approx -k_w \pm ik_w. \tag{27}$$

This simplification permits not to consider completely equations (24) and (25), but to replace them by simpler analogs, which will be used next

$$\frac{d^4w(x)}{dx^4} + 4k_w^4 w(x) = \frac{1}{g_w(x)}\frac{p(x)}{D_r}. \tag{28}$$

$$g_w^2(x)\frac{d^4w(x)}{dx^4} + 4k_w^4 w(x) = \frac{1}{g_w(x)}\frac{p(x)}{D_r}. \tag{29}$$

Thus, roots of characteristic equation for (1) and (24) can be considered as the same, and the general solution of the differential equation (24) assume in the form of equation (5). Partial solution w_* will be researched for the load of the form $p(x) = p_0\alpha e^{x(\beta+\varepsilon_w/H_w)}$ that takes all the analyzed variants (i. e. uniform and exponential load with different values of the parameter ε_w.

For the partial solution it is

$$w_* = \frac{p_0\alpha e^{x(\beta+\varepsilon_w/H_w)}}{D_r\left[(\beta+\varepsilon_w/H_w)^4+4k_w^4\right]}. \tag{30}$$

A general solution of the differential equation (24) for the shell stiffened near the surface is function

$$w(x) = \frac{w_0\alpha e^{xs_w}}{1+0.25s_w^4/k_w^4}\left\{1-e^{-k_wx}\left[\left(1+\frac{s_w}{k_w}\right)\sin(k_wx)+\cos(k_wx)\right]\right\}, \tag{31}$$

where $s_w = \beta+\varepsilon_w/H_w$ is the auxiliary constant inserted for cancellation.

Considering that s_w/k_w<<1 and s^4_w/k^4_w<<1, the equation (31) can be represented in the form

$$w(x) = w_0\alpha e^{xs_w}\left\{1-e^{-k_wx}\left[\sin(k_wx)+\cos(k_wx)\right]\right\}. \tag{32}$$

The similar method can be used to determine the deflections of shell, the load on which is circumscribed by Yansen-Kenen exponential law or on other conditions by the abutment of the shell to the surface. Thus, when the wall thickness is changed, obtained formulas of the Table 1 can be successfully used with only one difference – the result must be multiplied by coefficient $K_\varepsilon(x) = e^{x\varepsilon_w/H_w}$.

Comparative calculations of shells with constant and variable thickness of wall indicate that more accurate calculation practically does not influence on quantity of bearing internal efforts (moments and transverse forces), but displays only on diagram of deflections. Nature of the given influence is illustrated in the Figure 2, it can be noticed that disregard of functional dependence $t_{ef,r}(x)$ can lead to underestimation of shell deflections. Expansion of the given error is influenced by increase of thickness differences of shell near its surface and near the top. Il corresponds to relatively large parameter value ε_w>1.0.

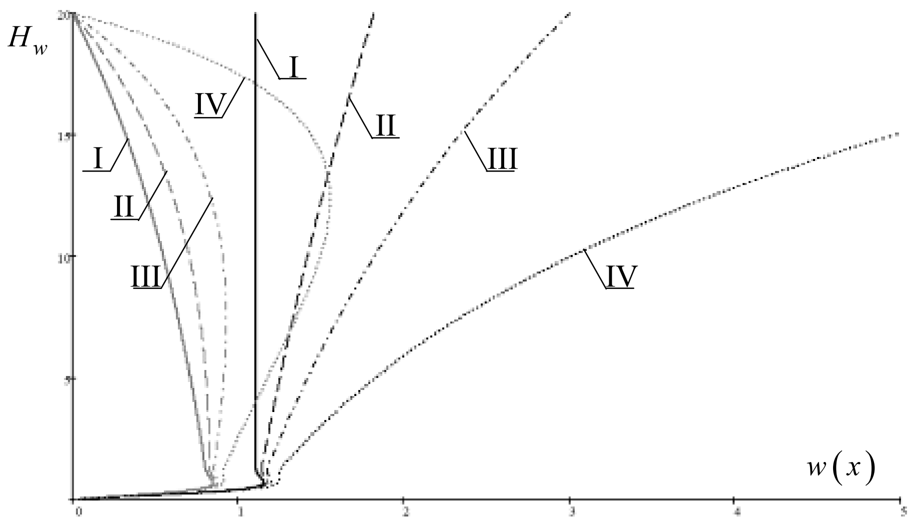

Figure 2 – Charts of deflections for shell with a stiff (swing joint) abutment to the surface:
I – ε_w= 1.0; II – ε_w= 0.5; III – ε_w= 1.0; IV – ε_w= 2.0;
black line – uniform load; gray line – exponential load

It should be mentioned that for Yansen-Kenen exponential load ordinate of maximum deflection of the shell $x_{w,max}$ is displaced to its top, when ε_w is increased. So, gripe conditions l have less influence on numeric evaluation. To obtain formula $x_{w,max}$, the Table 1 is used (p. 4 or 10). Let's differentiate any of these equations and equal the result to zero. Neglecting all the terms that are responsible for the behavior of the deflection function near the surface and liken the similar, we can obtain the next concise result; taking into account that parameter of the load α is equal to $\exp(-\beta H_w)$

$$x_{w,\max} = \frac{1}{\beta}\ln\left[\frac{\varepsilon_w \exp(\beta H_w)}{\varepsilon_w + \beta H_w}\right], \tag{33}$$

where the parameter β is determined by other equation of formulas (12) and is the function of the friction coefficient f_g and the lateral friction λ_0.

The formula indicates that $x_{w,max}$ depends on shell height H_w and diameter D_w, characteristics of the bulk material (parameters f_g and λ_0) and the nature of shell thikness changing in height (parameter ε_w). If to insert the obtained value $x_{w,max}$ in an equation from the Table 1 (p. 4 or 10), it can be determined maximum deflection of the shell that corresponds to accepted distribution of shell thickness in height.

In clear mathematic formulation the solution of differential equation (29) of shell with flat wall has considerable analytical difficulties. If to admit that function *w(x)* is not very different from its analog for shell with corrugated wall (in adjustment of cylindrical stiffness and parameter k_w), then to find a solution is easy enough and it corresponds to the circumscribed assumption.

So, formulas in the Table 1 stay relevant, considering multiple by coefficient $K_\varepsilon(x) = e^{x\varepsilon_w/H_w}$ considering distribution of thicknesses in height.

Conclusion.

1. Analytical dependences (Table 1) for deflection analysis, running bending moments and running transverse forces in shells with different abutment to the basis are obtained. the basic function of deflection and maximum value of relevant parameters of reaction have been shown.
2. The deflections of shells are relatively little influenced by shaping of sheets under the axially symmetric load and has an indirect meaning have been confirmed theoretically. But the quantities of bending moments for shells with profiling sheets are bigger in k_λ^2 times. And the quantities of transverse forces are bigger in k_λ^4 times.
3. Considering efforts from the temperature differential Δt, formula for defining the load p_t on steel shells has been obtained.
4. Dependence of internal efforts and displacements for shells with changeable wall thickness represented in analytical and graphical forms have been established.
5. Comparative calculations of shells with constant and variable thickness of the wall that correspond to formulas from the Table 1 have been made, considering multiple coefficient $K_\varepsilon(x) = e^{x\varepsilon_w/H_w}$. Distribution of thicknesses in height has been considered.

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