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Improved calculation method of reinforced concrete elements strength on inclined sections

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Application boundaries of the truss analogy and disk model methods to the strength calculation of reinforced concrete flexural elements inclined sections are established. Areas of structures failure by virtual compressed element (inclined strip) and compressed zone over dangerous inclined crack under the shear force are determined. The criterion of minimum limit force, which is perceived by the elements, is applied. Influence of concrete class, relative shear span and transverse reinforcement intensity on elements strength based on variational method in plasticity theory is specified. The data concerning the values of the transverse reinforcement coefficient at the boundaries of the failure from shear within the inclined strip and compressed zone over the dangerous crack are obtained.

Keywords: truss analogy, disk model, boundary of failure cases realization, minimum value of limit force, coefficient of transverse reinforcement

Удосконалена методика розрахунку міцності залізобетонних елементів за похилими перерізами

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Встановлені межі застосування методів фермової аналогії та дискової моделі для розрахунку міцності за похилими перерізами залізобетонних елементів, що згинаються. Визначені області реалізації руйнування залізобетонних конструкцій за умовним стиснутим елементом (похилою смугою) на дію поперечною сили та стиснутою зоною над небезпечною похилою тріщиною на основі застосуванням критерію мінімуму граничного зусилля, що сприймається елементами. Розрахункові залежності приведені до єдиної основи. Отримані дані щодо значень коефіцієнта поперечного армування елементів на межі руйнування від зрізу похилої смуги та стиснутої зони над небезпечною тріщиною. Уточнено вплив класу бетону, відносного прольоту зрізу та інтенсивності поперечного армування на міцність елементів, що підтверджується експериментальними дослідженнями. Для інтегрального оцінювання факторів впливу розглянуто задачу міцності похилої призми, завантаженої на торцях стиснутою нормальною та дотичною силами. Бетон розглядається як жорстко-пластичне тіло. Локалізація пластичної деформації в тонких шарах на поверхні руйнування є характерною для граничного стану бетону при зрізі. Аналіз отриманих варіаційний методом у теорії пластичності результатів дозволяє внести обгрунтовані зміни до визначення міцності стиснутого похилого елемента. Уточнена методика розрахунку міцності залізобетонних конструкцій за похилими перерізами на дію поперечної сили дозволяє отримати більш ефективні конструктивні рішення. Для інженерних розрахунків міцності запропонована залежність щодо визначення коефіцієнта поперечного армування ділянок конструкцій біля опор, який відповідає межі розглянутих випадків руйнування за похилим елементом та стиснутою зоною над небезпечною похилою тріщиною. Встановлена перспективність застосування теорії пластичності для вдосконалення конструктивних рішень згинальних залізобетонних елементів на основі подальшого уточнені міцності похилого елемента як складової фермової аналогії, стиснутої зони над небезпечною тріщиною і системного дослідження зрізу.

Ключові слова: фермова аналогія, дискова модель, межа реалізації випадків руйнування, мінімальне значення граничних зусиль, коефіцієнт поперечного армування

Introduction

Flexural reinforced concrete structures are widespread in practice and largely determine the construction cost. One way to optimize their design solutions is to improve calculation methods. At the same time an important place is occupied by the strength of the structures sections near the supports, which destruction on inclined sections has externally fragile avalanche character.

The theory of calculating the inclined sections strength, as well as the general theory of calculating reinforced concrete, has undergone several well-known development stages: «classical» theory, which used the basics of materials strength and considered the second stage of stress-strain state, in which the calculation was carried out at the main tensile stresses; the stage of calculating the elements by the destruction stage based on the truss analogy and the disk model; ideas deepening period about the stress-strain state of elements in the inclined cracks area.

The «classical» theory did not enable to consider the specific behavior of reinforced concrete elements in the stage of destruction. Thus, in the presence of transverse reinforcement, the actual value of the failure load exceeded the theoretical value considerably, and in its absence the calculation significantly underestimated the strength. The proposed later truss analogy considers reinforced concrete structures as trusses. They are divided into two chords – compressed one and tensile one with a constant arm of internal couple of forces along the element length, connected by an open gating of tensile braces of transverse reinforcement and compressed virtual concrete braces. The calculations provided full perception by reinforcement of the tensile force and by compressed force concrete. Based on the disk model, two schemes for the destruction of an inclined section were considered – under the bending moment and shear force action. Each scheme was described by one equilibrium equation of internal efforts and external forces. The efforts were considered to be perceived by concrete over dangerous crack and transverse reinforcement or pitch reinforcement. The method of limit equilibrium in inclined sections was the basis of the previous norms and remained normative for a long time.

During the period of in-depth research, the truss analogy method and the disk model were improved; the influence of the determining factors was specified. [1-12].

Review of research sources and publications

After enforcement of the normative documents DBN B.2.6-58:2009 [13] and DSTU B B.2.6.156: 2010 [14], the truss analogy is taken as a principle for calculation. Meanwhile, numerous experimental studies have shown the compressed inclined strip failure – the truss analogy element, as well as a concrete shear in a compressed zone over a dangerous inclined crack [1, 2, 7-10]. It should be noted that the virtual compressed element is crossed by transverse reinforcement that increases its strength, which decreases with decreasing inclination angle of the strip [13, 14]. Concerning the elements strength by shear of the compressed zone concrete, it should be noted that it is defined as the sum of the forces perceived by the compressed zone concrete and the transverse reinforcement that crosses the inclined crack [15]. With the relative shear span increase, the component of the shear effort perceived by concrete decreases. For the realization of a particular failure case, the transverse reinforcement intensity of arears near supports is crucial [7, 10].

Not solved earlier parts of the general problem are the demarcation of the above-mentioned cases of reinforced concrete elements destruction and the influences justified clarification of the determining strength factors: concrete strength, relative shear span, the transverse reinforcement intensity.

Objective of the work and research methods

It is determination of application areas of the truss analogy and disk model methods to the reinforced concrete elements strength calculation on inclined sections using the upper estimation of the efforts level and the variation method in the theory of plasticity.

Basic material and results

According to the current norms [13, 14], the force value $V_{Rd,1}$, which is perceived by the reinforced concrete element in the inclined section, is taken as smaller value between the values perceived by the concrete $V_{Rd,max}$ and the transverse reinforcement $V_{Rd,s}$.

For using qualities of concrete and reinforcement full-scale while designing reinforced concrete flexural structures it is advisable to apply the equation

$$V_{Rd,\max} = V_{Rd,s}\,, \tag{1}$$

where $V_{Rd,\max} = \dfrac{0.6 f_{cd} b_w z}{\cot\theta + \tan\theta}$, $V_{Rd,s} = \dfrac{A_{sw}}{s} z f_{ywd} \cot\theta$,

here f_{cd} – design value of concrete compressive strength, b_w – the element cross-section width, z – arm of internal couple of forces, θ – the the virtual compressed element inclination angle, A_{sw} – the cross-section area of shear reinforcement, s – step of shear reinforcement, f_{ywd} – design yield of shear reinforcement.

When $V_{Rd,s} > V_{Rd,\max}$ the transverse reinforcement is not effectively used and exceeding by the coefficient of reinforcement $\rho_w = \dfrac{A_{sw}}{b_w s}$ the value that is equal to

$$\rho_w = \frac{V_{Rd,\max}}{f_{ywd} b_w z \cot\theta}\,, \tag{2}$$

leads to reinforcement overrun.

In the case of a significant decrease in the transverse reinforcement intensity, which is characteristic for the structures designed in accordance with [15], or which have corrosive damage, the elements strength estimated according to [14] in the inclined sections is found to be much lower than that established experimentally [7, 10, 11]. It should be taken into consideration that the destruction of the areas near the supports occurs by shearing concrete in the most intense area within the compressed inclined element. The transverse reinforcement crosses this element along its entire length, that is, the tensile brace passes through the compressed brace and reinforces it.

Therefore, when establishing the application areas of the truss analogy method, the inclined compressed element strength ($V_{Rd,max}$) must be compared with the strength ($V_{Rd,2}$) determined by the disk model using the limit equilibrium conditions over the entire interval of inclination angles θ proposed in [14]. Thus, by $\theta = 45^{o}$ $V_{Rd,max}$ is almost equal to the force given in [15] in an inclined concrete strip by the shear force action.

In the interval of $1 \le \cos\theta \le 2.5$ for the calculated value of the shear force should be taken smaller from the values $V_{Rd,max}$ and $V_{Rd,2}$.

Shear form of destruction is characterized for concrete and reinforced concrete elements by action of the shear forces [16, 17]. It is also implemented within an inclined strip and a compressed zone over a dangerous inclined crack [18, 19].

For comparative analysis the disk model dependences [15] are corrected to the parameters used in [14] and take the form:

$$V_{Rd,2} = V_c + V_{sw}\ , \qquad (3)$$

here $V_{Rd,2}$ – shear effort perceived by the element in an inclined section, V_c and V_{sw} – efforts perceived by concrete and transverse reinforcement accordingly (fig. 1); and these components of equation (3) are equal

$$V_c = \frac{2 f_{ctd} b_w z^2}{c\eta^2}, \qquad (4)$$

$$V_{sw} = q_{sw} c_0 = q_{sw}\sqrt{\frac{2 f_{ctd}}{\rho_w f_{ywd}}}\,\frac{z}{\eta}, \qquad (5)$$

where f_{ctd} – design value of axial tensile strength of concrete, $c = 1.5 z \cot\theta$ – the inclined section projection length on the element longitudinal axis is assumed to be equal to the distance from the support to the concentrated force F or $c = \sqrt{\frac{2 f_{ctd} b_w}{q}}\,\frac{z}{\eta}$ by uniform distributed load q, $q_w = f_{ywd} A_{sw} / s$ – effort in the reinforcement per unit length of the element, coefficient $\eta = z/d$ (according to [14] is equal to 0,9, here d – the effective height of the cross-section of the element).

In this case, the calculation provides for the fulfillment of conditions: $z/\eta \le c_0 \le 2z/\eta$ and $c_0 \le c$.

The transverse reinforcement coefficient, which corresponds to the application boundary of calculation methods by truss analogy and disk model, is determined by the condition $V_{Rd,1} = V_{Rd,2}$.

The values of ρ_w by the class of reinforcement A400C and the corresponding value of cot θ are given in the table 1.

The dependence $V_{Rd} = \min(V_{Rd,1}, V_{Rd,2})$ on $\cot\theta$ and concrete class are given in fig. 2.

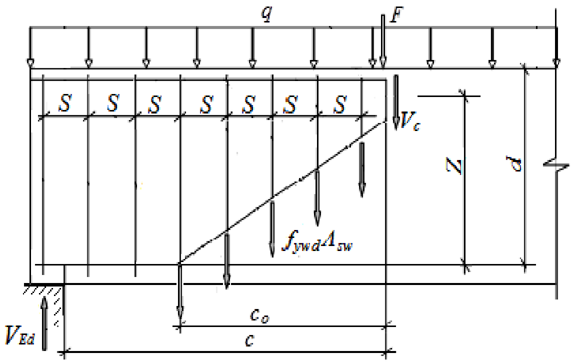


Figure 1 – Scheme of calculated forces on the shear force action on an inclined crack

By the value of cot θ, which exceeds that specified in table 1, the calculation of the shear force strength is made by the formula (3), and otherwise – by the truss analogy.

Table 1 – The values of the relative ultimate effort $\frac{V_{Rd}}{f_{cd} b_w z}$, cot θ and ρ_w at the boundaries of the failure cases at the inclined strip and the compressed zone over the dangerous crack

Class of concrete	$\frac{V_{Rd}}{f_{cd} b_w z}$	cot θ	ρ_w x10^3
C8/10	0.249	1.87	2.53
C16/20	0.272	1.57	6.29
C25/30	0.275	1.53	9.73
C32/40	0.277	1.49	12.9
C40/45	0.279	1,47	16.5
C50/60	0.279	1.47	19.8

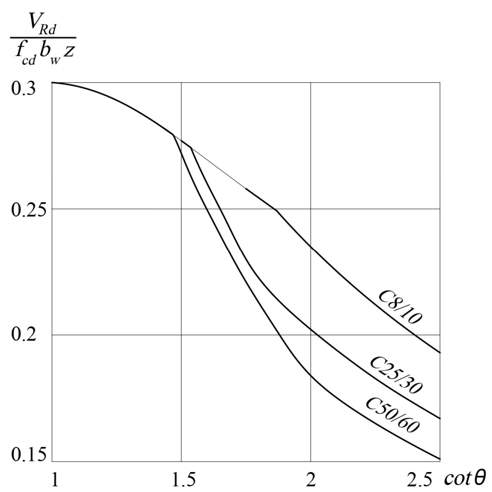


Figure 2 – The dependence of relative transverse shear effort $V_{Rd}/(f_{cd} b_w z)$ on cot θ

It should be noted that the compressed inclined strip strength, as a truss analogy virtual element, is influenced by the vertical or pitch reinforcement that crosses the strip. It increases its strength so much the greater projection strip length on the element longitudinal axis is the smaller the inclination angle to the horizontal θ is. The quantitative parameters of this influence are obtained from the strength problem solution of an inclined prism loaded by normal N and tangent T forces applied at the end surfaces of the prism as shear force components. The concrete prism collapses from shear at the surface of plastic deformation localization at an angle γ to the lateral faces. The reinforcement effect is considered by applying the forces q_w in the reinforcement per unit length of the prism lateral faces (fig. 3), which level is determined by data [20, 21], experimental studies materials [1, 2, 10] considering the ratio of reinforcement and concrete modules and the reinforcement location (the angle of its prism intersection).

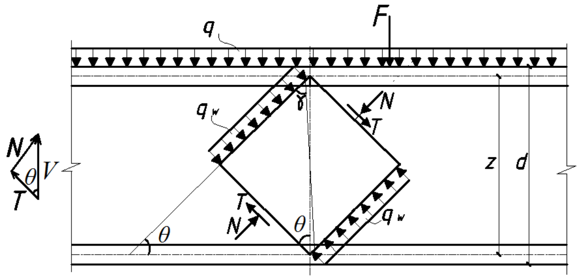


Figure 3 – The design scheme of the inclined prism

The plasticity theory is successfully used to solve the shear strength problems [22, 23]. Determination of the ultimate force perceived by the prism is carried out by a variational method using the virtual velocities principle [24, 25]. Discontinuous solutions of the prism strength problem are obtained. In this case, concrete is considered as a rigid-plastic body [26, 27]. As a plastic potential, the concrete strength condition is accepted [28]. The concrete prism strength decreases with decreasing angle of inclination θ, which is a consequence of the tangent component action T of the shear force. This decrease is offset by the transverse reinforcement effect mentioned above. Thus, the shear force, which is perceived by a virtual compressed inclined element at an interval $1 \le \cot\theta \le 2.5$, is proposed to be determined by the formula

$$V_{Rd,\max} = 0.3 f_{cd} b_w z \, . \tag{6}$$

Meanwhile, the ultimate load value by shear of the compressed zone over the dangerous inclined crack depends significantly on the transverse reinforcement intensity and the relative shear span (table 2).

The values of the transverse reinforcement coefficient at the boundaries of failure cases are given in table 3.

For better visualization, the strength calculation results are presented in fig. 4.

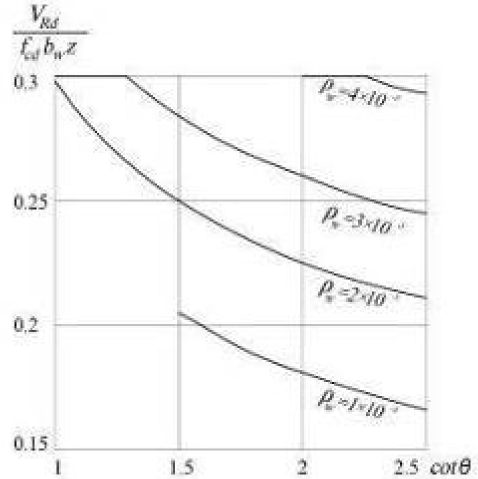
Table 2 – The value of the relative ultimate effort $\frac{V_{Rd}}{f_{cd} b_w z}$ by shear of the compressed zone over the inclined crack

ρ_w x10^3	$\frac{V_{Rd}}{f_{cd} b_w z}$ at cot θ			
	1	1.5	2	2.5
Concrete class C8/10				
1	0.225	0.205	0.181	0.173
2	0.298	0.25	0.225	0.211
3	0.333	0.284	0.26	0.245
4	0.381	0.332	0.308	0.293
Concrete class C16/20				
6	0.308	0.266	0.246	0.233
7	0.338	0.297	0.276	0.264
8	0.369	0.327	0.307	0.294
Concrete class C25/30				
8	0.282	0.243	0.224	0.212
9	0.302	0.264	0.244	0.233
10	0.323	0.284	0.265	0.253
11	0.344	0.305	0.286	0.274
12	0.365	0.326	0.306	0.295
Concrete class C32/40				
12	0.297	0.262	0.244	0.234
14	0.329	0.294	0.276	0.266
16	0.361	0.326	0.308	0.298
Concrete class C40/50				
16	0.304	0.271	0.255	0.245
18	0.33	0.297	0.28	0.27
20	0.356	0.322	0.305	0.296
Concrete class C50/60				
16	0.27	0.237	0.22	0.211
18	0.292	0.258	0.242	0.232
20	0.313	0.28	0.263	0.253
22	0.334	0.301	0.284	0.274
24	0.356	0.322	0.306	0.296

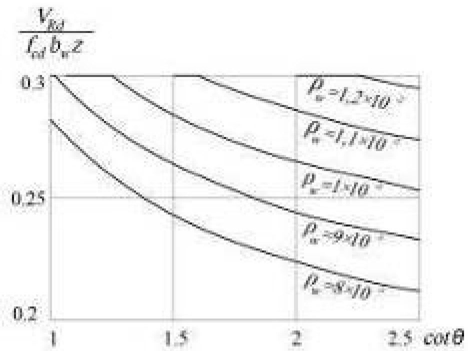
Table 3 – The value of the transverse reinforcement coefficient ρ_w at the boundaries of the destruction on the inclined strip and the compressed zone over the dangerous crack

$\rho_w \times 10^3$ at cot θ			
1	1,5	2	2,5
2.04	3.45	3.87	4.12
5.75	7.1	7.78	8.18
8.88	10.8	11.7	12.2
12.2	14.4	18.2	21.9
15.7	18.2	19.5	20.9
18.8	21.9	23.5	14.4

a)



b)



c)

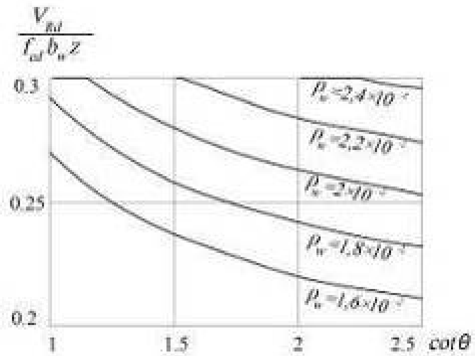


Figure 4 – Dependence of relative shear effort V_{Rd} / ($f_{cd}b_w$z) on cot θ at concrete class C8/10 (a), C25/30 (b), C50/60 (c)

To select the calculation method for the reinforced concrete element on the inclined sections for the shear force action when solving the problem of checking the strength (for the given concrete classes and reinforcement, the rectangular cross-section dimensions, the percentage of the transverse reinforcement ρ_w, the parameter z and the compressive element inclination angle θ) the ultimate value of the transverse reinforcement coefficient is established

$$\rho_{wR} = \frac{(1-2.5\chi)}{75\chi}\frac{f_{cd}}{f_{ywd}}\cot^{\alpha}\theta, \qquad (7)$$

where $\chi = \dfrac{f_{ctd}}{f_{cd}}$, $\alpha = \chi(1+75\chi)$.

If $\rho_w < \rho_{wR}$ the force V_{Rd} is equal to the sum of the forces in the compressed zone concrete V_c and in the transverse reinforcement V_{sw}, which are calculated by formulas (4 and 5) and determine the element strength by the inclined crack in the force action V_{Ed}.

In this case, if $V_{Ed} > V_c + V_{sw}$ the flexural element failure occurs by shearing the compressed zone concrete and achieving stresses in the transverse reinforcement, which crosses the inclined crack, the yield strength $A_j = s_k b_{mon}$.

If $\rho_w \geq \rho_{wR}$ – the force V_{Rd} is equal to $V_{Rd,\max}$, which is calculated by the formula (6) and determines the inclined element strength (providing $V_{Ed} > V_{Rd,\max}$ the element destruction occurs from the shear within the inclined strip.

Further improvement of flexural reinforced concrete elements structural decisions and structures creation of equal strength by inclined and normal sections are largely connected with increase of calculation accuracy of their strength on inclined sections based on the systematic shear study as a destruction form and considering the influence of all the determining factors.

Conclusions

1. Analysis of reinforced concrete elements experimental data and results of strength calculations on inclined sections necessitates the improvement of their calculation method.
2. The boundary of elements failure cases realization from concrete shear is established within the inclined strip and the compressed zone over the dangerous crack.
3. Based on the inclined prism shear problem solution by variation method in the plasticity theory, the influence of concrete class, relative shear span and reinforcement intensity are specified.
4. Areas of truss analogy method application and a disk model are determined and the method of calculating the reinforced concrete elements strength on inclined sections is improved.
5. For engineering calculation, the dependence of the transverse reinforcement coefficient ρ_{wR} at the elements destruction boundaries along the inclined strip and the compressed zone over the dangerous crack on the strength characteristics f_{cd}, f_{ctd}, f_{ywd} and the inclination angle θ is proposed.

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