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## The masonry calculation strength under the vertical and horizontal loads combined action by the variational method in the plasticity theory

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Possible causes of destruction of masonry in buildings with bearing masonry walls under the combined effect of vertical and horizontal loads are considered. The diagonal shear is highlighted as a typical case for partitions under seismic impacts. Based on the analysis of the results of experimental studies of samples for skewing, as models of the operation of the partition, proposals are presented for a kinematically possible scheme of the masonry elements destruction, which is proposed as a base for the calculation. The strength problem was solved by the variational method in the theory of plasticity, taking into account both strength characteristics of the masonry, the dimensions of the samples, and the loading areas. The peculiarities of the masonry work are taken into account by using the appropriate strength condition. The influence on the strength of reinforcement of masonry partitions during diagonal splitting is considered

**Keywords:** partition, seismic impact, diagonal splitting, plastic strain.

## Розрахунок міцності кам'яної кладки при сумісній дії вертикального та горизонтального навантаження варіаційним методом у теорії пластичності

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Розглянуто можливі випадки руйнування кладки в будівлях із несучими кам'яними стінами при сумісній дії вертикальних і горизонтальних навантажень. Виділено діагональний зсув як характерний випадок для простінків при сейсмічних впливах. Наголошено на відсутності нормативної методики розрахунку міцності для даного випадку руйнування. Використана класифікація тріщин в цегляних стінах в якості критерія реалізації окремих випадків руйнування простінків. На основі аналізу результатів експериментальних досліджень зразків на перекіс, як моделей роботи простінків, надані пропозиції щодо кінематично можливої схеми руйнування кам'яних елементів, котру запропоновано як базову для розрахунку. Задачі міцності розв'язані варіаційним методом у теорії пластичності з урахуванням обох характеристик опору кладки, розмірів зразків та площадок завантаження. Кам'яна кладка розглядається як жорстко-пластичне тіло. Пластична деформація зосереджена у тонких шарах на поверхні зсуву, а сусідні зони (диски) вважаються абсолютного жорсткими. Особливості робота кам'яної кладки враховано шляхом використання відповідної умови міцності. Умова міцності на ділянках зсуву в стиснутих областях розглядається як пластичний потенціал. Застосовується принцип віртуальних швидкостей. Задачі міцності розв'язуються шляхом дослідження функціоналу принципу на стаціонарний стан. Варіюються напрямки переміщення жорстких дисків та швидкості деформації. Отримані розрахункові залежності для визначення зусиль, котрі сприймають простінки цегляних стін у граничному стані. Граничне навантаження відповідає мінімуму потужності пластичної деформації. Встановлено вплив співвідношення між вертикальною і горизонтальною складовою навантаження на розміри площадки завантаження: при збільшенні величини горизонтальної сили розміри площадки завантаження зменшуються. Розглянуто вплив на міцність підсилення цегляних простінків при діагональному розколюванні

**Ключові слова:** простінок, сейсмічний вплив, діагональне розколювання, пластична деформація.



## Introduction

In Ukraine, a large number of buildings with bearing masonry walls are operated (designed). They are usually under the combined action of vertical and horizontal loads. Among the latter, seismic effects play the most significant role. According to [1], now about 15% of the territory of Ukraine is located in seismic zones with seismicity of more than 7 points.

The classification of cracks in masonry walls proposed in [2] under the combined action of vertical and horizontal loads can be used as a criterion for the implementation of individual masonry failure cases: shear in the horizontal plane, diagonal shear, destruction along the tensile zone; crushing.

The most typical damage to walls during seismic impacts is the formation of incline cracks that propagate both along with the stone and mortar and along the seams of the masonry (Fig. 1).



**Figure 1 – The destruction nature of a fragment of masonry walls under seismic influences according to [3]**

## Review of the research sources and publications

A significant amount of experimental research has been accumulated on the masonry walls operation on the combined action of vertical and horizontal loads, and have been carried out since the middle of the last century. In most cases, the samples were tested for a concentrated load applied along their diagonals (Figure 2, a). The choice of the test scheme is due to the analogy of the work of the masonry under seismic influences with the loading conditions, which are close to those arising in the frame when it is skewed (Fig. 2, b).

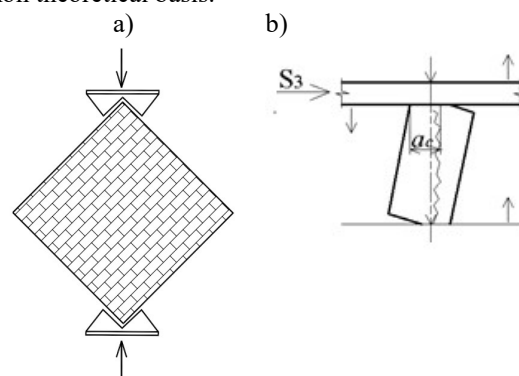
The nature of the failure is analyzed, determining factors of impact: masonry material, stone and mortar strength, internal and external reinforcement, reinforcement with soluble and concrete applications, cross and horizontal reinforced concrete strips, carbon fiber, diagonal metal straps, and others [4-17].

The calculation of the masonry for the action of the horizontal force in accordance with the national regulations [18] is carried out for shear (using the design shear resistance of the masonry  $f_{vd}$ ) and for bending in the corresponding direction (taking into account the characteristics of the tensile resistance in bending along the unbound section  $f_{sd1}$  – with the plane of cracking, parallel to the horizontal seams, and resistance to tensile bending along the tied section  $f_{sk2}$  – with the plane of cracking, perpendicular to the horizontal seams).

Combined, vertical and horizontal loads can be taken into account by applying the main eccentricities due to horizontal loads  $e_{hi}$  or  $e_{hm}$  when calculating the strength reduction factor  $\Phi$  or by applying an increased design tensile bending resistance of the masonry in a plane parallel to the horizontal seam (in an unbound section).

## Definition of unsolved aspects of the problem

Currently, there is no methodology for calculating masonry structures with the combined action of vertical and horizontal forces, which would be based on a common theoretical basis.



**Figure 2 – Test of masonry for distortion:**  
a – test scheme; b – the third stage of deformation of partition under seismic effects

## Problem statement

At the National University «Yuri Kondratyuk Poltava Polytechnic» a variational method of the theory of plasticity [19-22] is proposed as a basis for calculating masonry during diagonal splitting.

## Basic material and results

In the theoretical model, at the stage of failure, the partition is divided into four hard disks: two wedges under the load area (in the general case, the wedges should be non-equal-sided) and two hard disks, outlined by shear sections of the wedges and a splitting plane connecting their tops.

The wedges move towards each other; the other two hard drives move away from each other in a direction perpendicular to the splitting plane. There are four unknowns in the problem: two angles of inclination of the wedges shear sections to the vertical, the ratio of the speeds of hard disks movement, and the ultimate load (Fig. 3).

The frontal projection of the sample has a rectangular shape, i.e.,  $L \neq H$ , the dimensions of the load sections on the vertical and horizontal edges of the element differ from each other  $a_1 \neq a_2$ . Correspondingly, the angles of the seal wedge are  $\gamma_1 \neq \gamma_2$ .

The strength problem is solved in the following sequence:

1) at first, the speed jumps are sequentially found in the areas of the failure surface  $AC$ ,  $BC$ ,  $CC_1$  (Fig. 4), as well as the sizes of these areas:

- for  $AC$  and  $BC$  sections, speed breaks (Fig. 4) are calculated as:

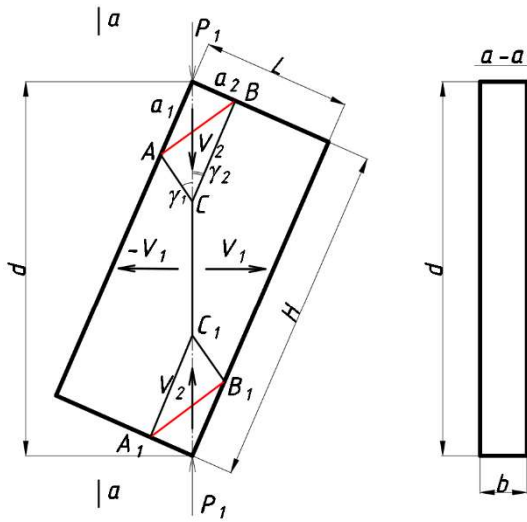


Figure 3 - Kinematic diagram of the failure of a masonry partition with diagonal splitting

$$\left. \begin{aligned} \Delta V_{n1} &= V_1 \cos \gamma_1 - V_2 \sin \gamma_1 \\ \Delta V_{t1} &= V_1 \sin \gamma_1 + V_2 \cos \gamma_1 \\ \Delta V_{n2} &= V_1 \cos \gamma_2 - V_2 \sin \gamma_2 \\ \Delta V_{t2} &= V_1 \sin \gamma_2 + V_2 \cos \gamma_2 \end{aligned} \right\}, \quad (1)$$

here  $\gamma_1$  and  $\gamma_2$  are the angles of inclination of the destruction areas of the  $AC$  and  $BC$  to the vertical plane (unknown parameters);

- surface areas, respectively:

$$\left. \begin{aligned} S_{AC} &= \frac{a_1 \sin \alpha_1}{\sin \gamma_1} b, \\ S_{BC} &= \frac{a_2 \sin \alpha_2}{\sin \gamma_2} b, \end{aligned} \right\} \quad (2)$$

where  $b$  is the sample thickness;

$$\sin \alpha_1 = \frac{\operatorname{tg} \alpha_1}{\sqrt{1 + \operatorname{tg}^2 \alpha_1}},$$

$$\sin \alpha_2 = \frac{\operatorname{tg} \alpha_2}{\sqrt{1 + \operatorname{tg}^2 \alpha_2}},$$

$$\operatorname{tg} \alpha_1 = \frac{L}{H}, \quad \operatorname{tg} \alpha_2 = \frac{H}{L};$$

- for section  $CC_1$  speed breaks

$$\left. \begin{aligned} \Delta V_n &= 2V_1 \\ \Delta V_t &= 0 \end{aligned} \right\} \quad (3)$$

and the surface area of the section  $CC_1$  is equal to

$$S_{CC1} = \frac{H}{\cos \alpha_1} - 2a_1 \left( \cos \alpha_1 + \frac{\sin \alpha_1}{\operatorname{tg} \gamma_1} \right); \quad (4)$$

2) the functional of the variational method is written, which includes three components:

- the power of plastic strain of masonry at the site of the  $AC$

$$m \left[ 2B \sqrt{1 + 0,25 \left( \frac{\Delta V_{t1}}{\Delta V_{n1}} \right)^2} - 1 \right] \cdot \Delta V_{n1} \cdot S_{AC}, \quad (5)$$

were  $m = f_b - f_{xd2}$ ,  $B^2 = (1 + \chi / (1 - \chi)^2) / 3$ ;  $\chi = f_{xd2} / f_b -$

the ratio of tensile strength to the compressive strength of masonry;

- the power of plastic strain of masonry at the aircraft site

$$m \left[ 2B \sqrt{1 + 0,25 \left( \frac{\Delta V_{t2}}{\Delta V_{n2}} \right)^2} - 1 \right] \Delta V_{n2} S_{BC}; \quad (6)$$

- the power of external forces at given speeds on sites  $CC_1$  and  $AB$  (section  $CC_1$  is taken as the main one, on which the ultimate tensile stresses act, the last component describes the action of external forces on section  $AB$ )

$$f_{xd2} V_1 S_{CC1} - f_{loc} V_2 S_{AB}. \quad (7)$$

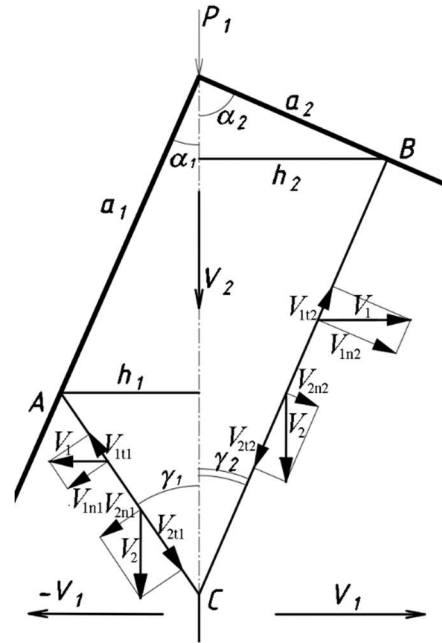


Figure 4 - Before determining the parameters at the sites of destruction

The expression is converted sequentially.

Section  $AC$ :

$$\begin{aligned} m \left[ 2B \sqrt{1 + 0,25 \left( \frac{\Delta V_{t1}}{\Delta V_{n1}} \right)^2} - 1 \right] \Delta V_{n1} S_{AC} = \\ = m \left[ 2B \sqrt{(k - \operatorname{tg} \gamma_1)^2 + 0,25 (k \operatorname{tg} \gamma_1 + 1) - (k - \operatorname{tg} \gamma_1)} \right] \times \\ \times \frac{b \alpha_1 \sin \alpha_1}{\operatorname{tg} \gamma_1}. \end{aligned}$$

Section  $CB$ :

$$\begin{aligned} m \left[ 2B \sqrt{1 + 0,25 \left( \frac{\Delta V_{t2}}{\Delta V_{n2}} \right)^2} - 1 \right] \Delta V_{n2} S_{CB} = \\ = m \left[ 2B \sqrt{(k - \operatorname{tg} \gamma_2)^2 + 0,25 (k \operatorname{tg} \gamma_2 + 1) - (k - \operatorname{tg} \gamma_2)} \right] \times \\ \times \frac{b \alpha_2 \sin \alpha_2}{\operatorname{tg} \gamma_2}. \end{aligned}$$

Section  $CC_1$ :

$$\frac{2f_{sd2}S_{CG}}{2}V_1 = f_{sd2}b \left( \frac{H}{\cos\alpha_1} - 2 \left( a_1 \cos\alpha_1 + \frac{a_1 \sin\alpha_1}{\operatorname{tg}\gamma_1} \right) \right) =$$

$$= f_{sd2}b \left( \frac{H}{\cos\alpha_1} - 2a_1 \left( \cos\alpha_1 + \frac{\sin\alpha_1}{\operatorname{tg}\gamma_1} \right) \right).$$

After substituting the components into the functional, and taking into account its equality to zero, a formula was obtained for determining the ultimate load as a function of unknown parameters:

$$\frac{P}{mb} = R_1 + R_2 + \frac{f_{sd2}}{m} \left( \frac{H}{\cos\alpha_1} - 2a_1 \left( \cos\alpha_1 + \frac{\sin\alpha_1}{\operatorname{tg}\gamma_1} \right) \right), \quad (8)$$

where:

$$R_1 = \left[ 2B\sqrt{(k-\operatorname{tg}\gamma_1)^2 + 0,25(k-\operatorname{tg}\gamma_1+1)^2} - (k-\operatorname{tg}\gamma_1) \right] \times$$

$$\times \frac{a_1 \sin\alpha_1}{\operatorname{tg}\gamma_1},$$

$$R_2 = \left[ 2B\sqrt{(k-\operatorname{tg}\gamma_2)^2 + 0,25(k-\operatorname{tg}\gamma_2+1)^2} - (k-\operatorname{tg}\gamma_2) \right] \times$$

$$\times \frac{a_2 \sin\alpha_2}{\operatorname{tg}\gamma_2}.$$

The angles of inclination of the faces of the seal wedge are interconnected by the dependence

$$\operatorname{tg}\gamma_1 = \frac{\operatorname{tg}\alpha_1 \operatorname{tg}\alpha_2}{k_1 \operatorname{tg}\alpha_1 \operatorname{tg}\gamma_2 - \operatorname{tg}\gamma_2 + k_1}. \quad (9)$$

With symmetrical application of a load for a square sample, we use the kinematically possible scheme in Fig. 5.

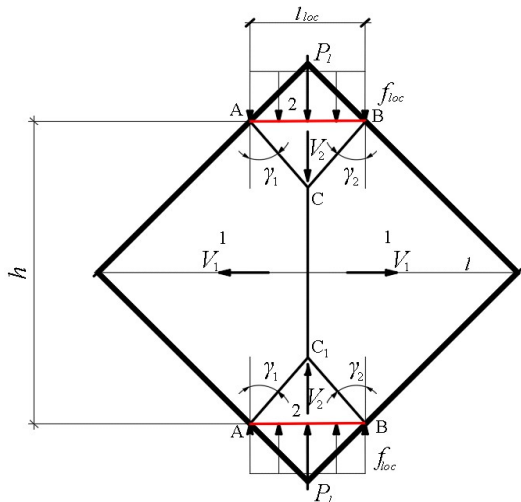


Figure 5 - Kinematically possible scheme of the failure of a square masonry specimen during diagonal loading

Formula (8) for determining the ultimate load takes the form

$$\frac{f_{loc}}{m} = \left[ \frac{2B\sqrt{(k-\operatorname{tg}\gamma)^2 + 0,25(k\operatorname{tg}\gamma+1)^2}}{(k-\operatorname{tg}\gamma)} - 1 \right] \times$$

$$\times \left( \frac{k-\operatorname{tg}\gamma}{\operatorname{tg}\gamma} \right) + \frac{f_{sd1}k(at\operatorname{tg}\gamma-1)}{\operatorname{tg}\gamma m},$$

here  $f_{loc} = P/b|_{loc}$ ,  $k=V_1/V_2$ , and  $\alpha=h/l|_{loc}$ .

During solving the strength problem, MS Excel and its add-on "Solver" are used: the objective function (10) is optimized from unknown parameters,  $\operatorname{tg}\gamma$ , and  $k$ .

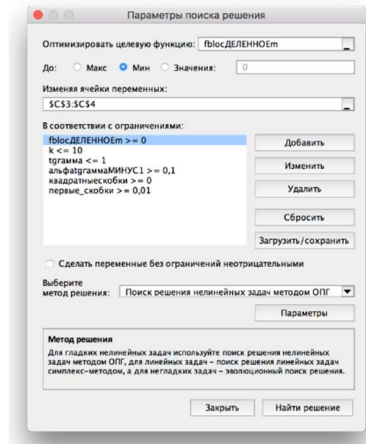


Figure 6 - Finding the unconditional minimum of function (10) with diagonal loading using a MS Excel

In the calculations of reinforced elements (for example, using carbon fiber tapes in three levels, they are considered as external reinforcement, while an additional term is introduced into the functional, taking into account the deformation power of carbon fiber at given speeds. The kinematically possible scheme of the failure of such an element is shown in Fig. 7).

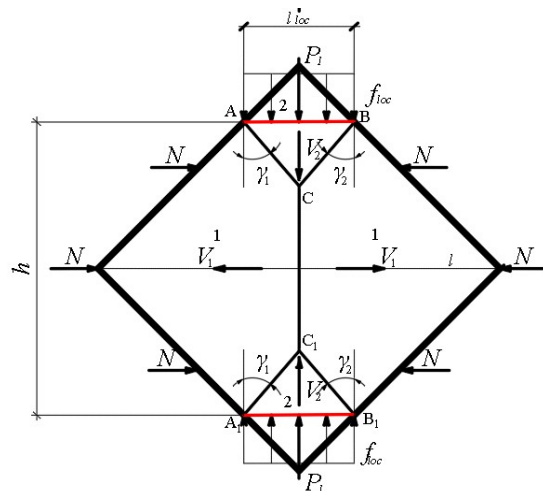


Figure 7 - Kinematically possible scheme of the failure of a square reinforced masonry specimen with diagonal loading

The ultimate load, in this case, is calculated as

$$\frac{f_{bl_{loc}}}{m} = \frac{\left[ 2B\sqrt{(k-tg\gamma)^2 + 0,25(1+ktg\gamma)^2} - (k-tg\gamma) \right]}{tg\gamma} + \frac{f_{x_{d2}}k(atg\gamma-1)}{mtg\gamma} + \frac{(2A_1+A_2)\sigma_y k}{Bl_{loc}m} = 0, \quad (11)$$

here  $A_1$  and  $A_2$  are the areas of the reinforcement strips,  $\sigma_y$  is the stress in the strips at the moment of failure.

In the proposed design schemes, it is possible to take into account the failure of the stone and mortar (along the diagonal plane) or only along with the mortar (curve, passing along vertical and horizontal seams) by using various characteristics of the tensile resistance of the masonry.

In addition, the ratio between the vertical and horizontal components of the load has a significant effect on the size of the loading area: with an increase in the value of the horizontal force, the size of the loading area decreases. At low values of the horizontal force, on the contrary, the dimensions of the sealing wedges increase, and the length of the separation section decreases, which usually leads to an increase in the ultimate load.

## Conclusions

During determining the bearing capacity of the partitions, the variational method of the theory of plasticity was used, developed at the National University "Yuri Kondratyuk Poltava Polytechnic" for the calculation of structures made of materials with a significant difference between the compression and tension strength. On the basis of the general provisions of the variational method, the specifics of the masonry work are taken into account. Based on the results of the analysis of the prototype's destruction nature on a concentrated load applied along the diagonals, kinematic schemes of destruction are proposed, which are the basis for calculating the masonry strength. The problem of the strength of masonry elements for diagonal splitting, which takes place in prototypes and walls of existing buildings and structures, has been solved. Both masonry characteristics were taken into account: compression and tension, sample dimensions, and the nature of the load application. The influence on the strength of reinforcement of masonry walls during diagonal splitting is considered, which can be used during the operation and reconstruction of the building and are given in the corresponding section of the design documentation.

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